Event Risk Premia and Non-convex Volatility Smiles

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Introduction

- Discrete events such as elections or macroeconomic announcements create risks in the prices of financial instruments
- Firm-specific event risk can be diversified, but systemic risk cannot and will be priced by the market
- Index returns on such days are higher on average than on normal days
- One goal of this paper: Quantify event risk premium in an expected utility framework
- In our model, non-convex volatility smiles occur in the run-up to events
- Second goal of this paper: Price options on this event risk and analyze conditions under which non-convex volatility smiles arise and their relation to bimodality of RNDs

Literature: Event risk premium

- Part of the literature on event risk assumes (or studies cases where) there is no risk premium (Froot and Posner, 2002; Hanke, Stöckl, and Weissensteiner, 2020)
- *Macroeconomic announcement premium* (see the review of Ai, Bansal, and Guo, 2023): positive average excess returns and higher volatility on event days, i.e. when news on interest rates, inflation, unemployment, or other key economic indicators are published (Savor and Wilson, 2013a,b; Lucca and Moench, 2015; Wachter and Zhu, 2022).
- Liu and Shaliastovich (2023) document positive average excess returns of 50bp on the day after U.S. elections
- Can also be extracted ex ante from the differences in the prices of options expiring shortly before/after the event (Liu, Tang, and Zhou, 2022; Knox et al., 2024)
- Among the papers from this strand of literature, our model is most closely related to Liu, Tang, and Zhou (2022)

Literature: Non-convex volatility smiles

- Dubinsky et al. (2019) analyze event risk effects in option prices in a model with stochastic, mean-reverting diffusive volatility together with both event-induced jumps and jumps that may occur randomly at any time between economic events
- Alexiou et al. (2023) extend Dubinsky et al. (2019) and replace the normality assumption for the deterministically-timed jumps by a mixture of two normals, which may exhibit bimodality. Major findings: concavity in volatility smiles signals event risk, and in around 80% of all cases of concave volatility smiles, they also observe bimodal risk-neutral densities
- Glasserman and Pirjol (2023) derive bounds on the number of crossings of the implied volatility function with a fixed level. They find that in general, there is no simple relation between the number of modes of the risk-neutral density and the shape of the implied volatility smile
- Among the papers from this strand of literature, our model is most closely related to Alexiou et al. (2023)

Basic setting

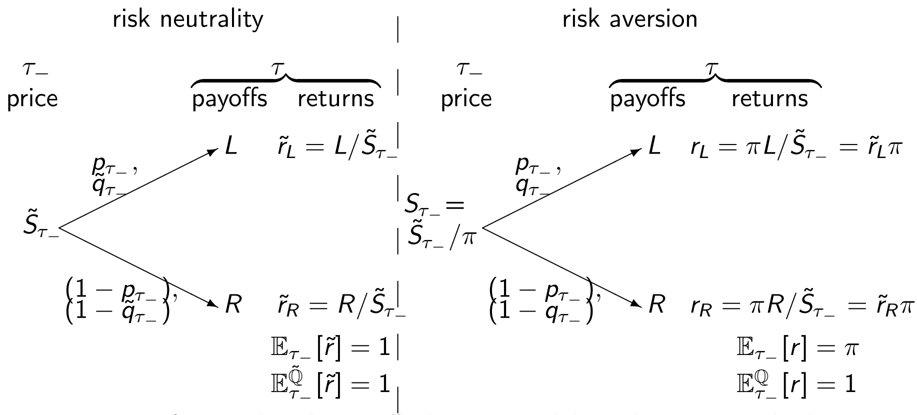

Figure: Basic setting: Outcome-dependent payoffs, their prices, and the resulting returns under the assumptions of risk neutrality (left panel) and risk aversion (right panel). Quantities that relate *solely* to the case of risk neutrality are indicated by a tilde. Expectations are taken with respect to the real-world probability measure $\mathbb{P}$ unless explicitly stated otherwise.

Basic setting with conditional return densities

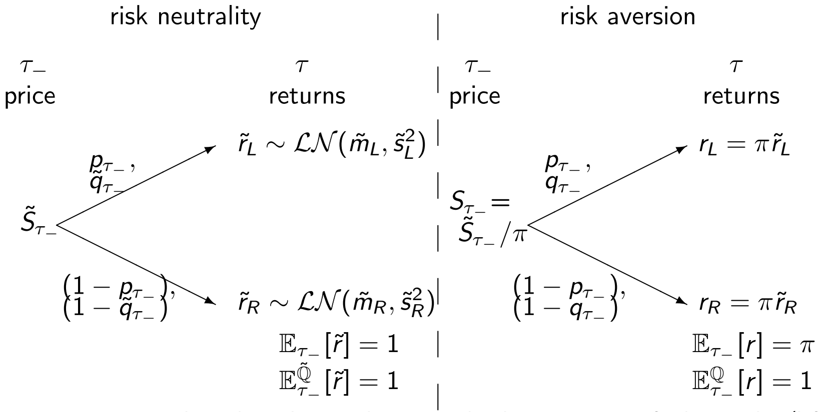

Figure: Basic setting with conditional return densities under the assumptions of risk neutrality (left panel) and risk aversion (right panel). Quantities that relate *solely* to the case of risk neutrality are indicated by a tilde. Expectations are taken with respect to the real-world probability measure $\mathbb{P}$ unless explicitly stated otherwise.

Risk premia in an expected utility framework

- Assumptions: risk-averse representative investor who decides based on expected utility in returns, $U(\cdot) = \mathbb{E}[u(\cdot)]$, with a generic von Neumann-Morgenstern utility function, $u(r)$ that is increasing and concave
- Four different cases which are given by the combination of deterministic/stochastic conditional event returns (viewed at time τ_-) with deterministic/stochastic event outcome probabilities (when moving to time points $t < \tau$)
- At time τ_-, in equilibrium the expected return π satisfies

$$p_{\tau_-} u(\tilde{r}_L \pi) + (1 - p_{\tau_-}) u(\tilde{r}_R \pi) = u(1), \quad \tilde{r}_L \geq 1 \geq \tilde{r}_R \tag{1}$$

for deterministic event outcomes, and for conditional densities we get

$$p_{\tau_-} \int_0^\infty u(\tilde{r}_L \pi) f_{\mathcal{LN}}(\tilde{r}; \tilde{m}_L, \tilde{s}_L) \mathrm{d}\tilde{r} + (1 - p_{\tau_-}) \int_0^\infty u(\tilde{r}_R \pi) f_{\mathcal{LN}}(\tilde{r}; \tilde{m}_R, \tilde{s}_R) \mathrm{d}\tilde{r} = u(1), \tag{2}$$

where the parameters $\tilde{m}_L$, $\tilde{m}_R$, $\tilde{s}_L$, and $\tilde{s}_R$ ensure that $\tilde{\mu}_L \geq 1 \geq \tilde{\mu}_R$ and $\mathbb{E}[\tilde{r}] = 1$

Risk premia in an expected utility framework /2

- At time $t < \tau$, for known event outcome probabilities p_{τ_-}, $\tilde{S}_t = \exp(-r(\tau_- - t))\tilde{S}_{\tau_-}$
- When the probabilities are allowed to change over time, since we assume the terminal payoffs (or their distributions) to be fixed, only the time t expectation of p_{τ_-}, $\mathbb{E}_t[p_{\tau_-}]$, is relevant
- From financial theory, we know that risk-neutral event outcome probabilities q_t themselves must be martingales under $\mathbb{Q}$: $\mathbb{E}_t^{\mathbb{Q}}[q_{\tau_-}] = q_t$.
- The Radon-Nikodym derivative, $\mathrm{d}\mathbb{P}/\mathrm{d}\mathbb{Q}$, is given by the ratios of the corresponding event outcome probabilities
- $\Rightarrow$ the process followed by p_t must be a martingale under $\mathbb{P}$:

$$\mathbb{E}_t^{\mathbb{Q}}[q_{\tau_-}] = q_t \quad \Longleftrightarrow \quad \mathbb{E}_t[p_{\tau_-}] = p_t$$

Risk premia in an expected utility framework /3

- At time $t < \tau$, the representative investor forms expectations about the prospective values of the event outcome probabilities at time τ.
- Final payoffs are fixed. Together with the martingale property of the process for p_t, this implies that equation (1) in the case of deterministic event returns becomes

$$p_t u(\tilde{r}_L \pi_t) + (1 - p_t) u(\tilde{r}_R \pi_t) = u(1), \tag{3}$$

- By the same arguments, we can extend equation (2) to the case of stochastic event outcome probabilities:

$$p_t \int_0^\infty u(\tilde{r}_L \pi_t) f_{\mathcal{LN}}(\tilde{r}; \tilde{m}_L, \tilde{s}_L) \mathrm{d}\tilde{r} + (1 - p_t) \int_0^\infty u(\tilde{r}_R \pi_t) f_{\mathcal{LN}}(\tilde{r}; \tilde{m}_R, \tilde{s}_R) \mathrm{d}\tilde{r} = u(1) \tag{4}$$

Examples for resulting formulas

- Quadratic utility,

$$u(r) = -0.5(\alpha - r)^2 \tag{5}$$

with $r < \alpha$, $t = \tau_-$, deterministic event outcomes:

$$\pi = \frac{\alpha - \sqrt{(\alpha - 1)^2 + \tilde{\sigma}^2(1 - 2\alpha)}}{\tilde{\sigma}^2 + 1} \tag{6}$$

- Power utility,

$$u(r) = \frac{r^{\overline{\gamma}} - 1}{\overline{\gamma}}, \tag{7}$$

$t < \tau_-$, conditional return densities:

$$\pi_t = \left(p_t \mathbb{E}_t\left[\tilde{r}_L^{\overline{\gamma}}\right] + (1 - p_t)\mathbb{E}_t\left[\tilde{r}_R^{\overline{\gamma}}\right]\right)^{-1/\overline{\gamma}} \tag{8}$$

with

$$\mathbb{E}_t\left[\tilde{r}_L^{\overline{\gamma}}\right] = \exp\left(\overline{\gamma}\tilde{m}_L + \frac{1}{2}\overline{\gamma}^2\tilde{s}_L^2\right) \tag{9}$$

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Option pricing in the world without a risk premium

- We start with the basic setting with conditional event return densities
- In a risk-neutral world, the option price at time τ_- is given by the mixture-of-lognormals model (see Ritchey (1990)), where each of the two components can be valued via a modification of the Black (1976) model
- Differences to the Black model:
  - Parameters do not scale with time to maturity
  - Non-zero location parameters of conditional densities
- For the time t price of a call on $\tilde{S}_\tau$ with strike K, we get

$$C_t(\tilde{F}_t, K, \tilde{m}_L, \tilde{s}_L, \tilde{m}_R, \tilde{s}_R) = \tilde{q}_t MBC(\tilde{F}_t, K, \tilde{m}_L, \tilde{s}_L) + (1-\tilde{q}_t) MBC(\tilde{F}_t, K, \tilde{m}_R, \tilde{s}_R), \tag{10}$$

where the modified Black call price, $MBC(\cdot)$, for generic $\tilde{m}$ and $\tilde{s}$, is given by

$$MBC_t(\tilde{F}_t, K, \tilde{m}, \tilde{s}) = \exp(-r(\tau - t))[N(d_+)\tilde{F}_t - N(d_-)K], \tag{11}$$

$$d_+ = \left(\ln(\tilde{F}_t/K) + \tilde{m} + \tilde{s}^2/2\right)/\tilde{s}, \tag{12}$$

$$d_- = d_+ - \tilde{s}. \tag{13}$$

Option pricing in the world with a risk premium

- In the risk-averse world with underlying S_t, all risk neutral event returns $\tilde{r}$ are multiplied by the risk premium π
- This corresponds to adding $\ln \pi$ to the log returns in the Black model
- Further, the risk-neutral event outcome probabilities change from $\tilde{q}$ to q in the presence of a risk premium with $\mathbb{E}_t^{\mathbb{Q}}[r] = 1$
- For the call price, this yields

$$C_t(\cdot, \pi_t) = q_t MBC(F_t, K, \tilde{m}_L, \tilde{s}_L, \pi_t) + (1 - q_t) MBC(F_t, K, \tilde{m}_R, \tilde{s}_R, \pi_t), \quad (14)$$

where the modified Black call price, $MBC(\cdot)$, for generic $\tilde{m}$ and $\tilde{s}$, is given by

$$MBC_t(F_t, K, \tilde{m}, \tilde{s}, \pi_t) = \exp(-r(\tau - t))[N(d_+)F_t - N(d_-)K], \quad (15)$$

$$d_+ = \left(\ln(F_t/K) + \tilde{m} + \tilde{s}^2/2 + \ln \pi_t\right)/\tilde{s}, \quad (16)$$

$$d_- = d_+ - \tilde{s}, \quad (17)$$

π_t is calculated from equation (8), and q_t is defined by $\mathbb{E}_t^{\mathbb{Q}}[\tilde{r}\pi] = 1$.

Empirical implications

1. Risk premia: For plausible values of relative risk aversion and other parameters, the risk premia resulting from the model are in the range documented in the empirical literature
2. Non-convexity of smiles: Simulations indicate that in our model, concavity of volatility smiles implies bimodality of RNDs (proof currently being worked on). Concave volatility smiles become more likely as
   - event outcome probabilities are closer to 0.5,
   - the distance between expected event returns increases,
   - the variances of conditional return distributions decrease, and
   - the time to maturity decreases.

Conclusion

- Simple model for event risk premia in an expected utility framework
- Combined with the well-known mixture-of-lognormals model, closed-form solutions for risk premia and option prices can be derived
- Application of the model to parameters of event return distributions estimated in previous work leads to magnitudes for event risk premia that are in line with the empirical literature
- The model features concave volatility smiles
- Bimodality of risk-neutral return distributions seems to be a necessary, but not a sufficient condition for concave volatility smiles