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Abstract

Purpose – The purpose of this paper is to create a universal (asset-class-independent) portfolio risk index for a global private investor.

Design/methodology/approach – The authors first discuss existing risk measures and desirable properties of a risk index. Then, they construct a universal (asset-class-independent) portfolio risk measure by modifying Financial Turbulence of Kritzman and Li (2010). Finally, the average portfolio of a representative global private investor is determined, and, by applying the new portfolio risk measure, they derive the *P*rivate investor *R*isk *I*nde*X*.

Findings – The authors show that this index exhibits commonly expected properties of risk indices, such as proper reaction to well-known historical market events, persistence in time and forecasting power for both risk and returns to risk.

Practical implications – A dynamic asset allocation example illustrates one potential practical application for global private investors.

Originality/value – As of now, a risk index reflecting the overall risk of a typical multi-asset-class portfolio of global private investors does not seem to exist.

Keywords Investment application, Multi-asset portfolio, Portfolio turbulence, Risk index

Paper type Research paper

1. Introduction

Globally, financial wealth is split roughly equally between institutional and (high-net-worth) private investors. Average asset allocations of institutional investors, however, are quite different from those of private investors[1]. Both have a demand for accurate and timely information on changes in the overall risk structure of their specific portfolios in relation to suitable benchmark portfolios.

Risk indices to capture changes in the risk structure of financial markets are numerous. However, existing risk indices either focus on just one asset class (e.g. equity volatility indices like the VIX) or are tailored mainly to the needs of policymakers and economists rather than investors[2]. Whereas institutional investors have both access to the relevant information and the resources to process it according to their specific needs, private investors frequently lack at least one of both. To the best of our knowledge, a benchmark risk index catering specifically to the needs of private investors with international and multi-asset-class investments does not yet exist.

Existing risk indices are based on classical risk measures, for example, volatility. Whereas this is appropriate for single-asset-class risk indices, it has its disadvantages when

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applied in a multi-asset-class context: first, classical risk measures are typically asset class-specific (volatility for equities, duration for fixed income, […]) and difficult or impossible to aggregate meaningfully. Second, they do not capture certain aspects of risk which can, however, be picked up by more sophisticated approaches.

Hence, instead of using classical risk measures for the construction of our risk index, we modify a different risk metric that has been suggested in the literature for our purpose. We prefer to call it a risk *metric* rather than a risk *measure*[3] for two reasons: first, it captures additional aspects of risk, such as unusualness of current correlations relative to normal levels. Second, our focus is not on a comparison of the theoretical properties of this metric to those of classical risk measures. In particular, we select this metric not because of superior theoretical properties when compared to classical risk measures but, rather, because it satisfies certain desirable criteria from an application-oriented point of view, such as transparency and easy availability. These criteria will be discussed in more detail in Section 2.

On our way to this goal, we follow a three-step approach. Any benchmark financial risk index consists of two basic building blocks – a risk metric and a representative (benchmark) portfolio of financial assets. As a first step, we discuss desirable properties of a multi-asset-class risk index. We describe a suitable risk metric based on a modification of *F*inancial *T*urbulence (FT) [originally proposed by Kritzman and Li (2010)] called *P*ort*F*olio *T*urbulence (PFT). This new risk metric overcomes the shortcomings of classical risk measures (e.g. volatility) described above.

In a second step, we construct a representative (benchmark) portfolio for global private investors (high-net-worth individuals [HNWI]) based on available data and call this the *PR*ivate *I*nvestor *P*ortfolio (PRIP). We will show that this portfolio differs significantly from the global market portfolio because typical portfolio compositions of private investors deviate from those of institutional investors. As an example, institutional investors hold substantially higher positions in money markets and fixed income securities, whereas private investors invest a higher fraction of their wealth into real estate.

In a final third step, we apply PFT to the PRIP to arrive at the *P*rivate investor *R*isk *I*nde*X* (PRIX). Hence, the PRIX is a benchmark index of the risk of the average portfolio of private (high-net-worth) investors. In principle, PFT could be used to indicate risk for all types of portfolios, especially the specific portfolio of each private investor and so enable him to compare the risk of his portfolio both across time and to a benchmark portfolio like the PRIP. In this paper, however, we mainly focus on its use as the basis for the PRIX. Calculating the PRIX on historical data, we show that it incorporates all the desirable criteria described in Section 2.

The paper is structured as follows. Section 2 discusses classical risk measures and desirable features of the PRIX. In Section 3, we construct the PRIX, starting with deriving the PFT measure used as its basis. As a second component, we describe our benchmark portfolio PRIP, by drawing on information from Cap Gemini's World Wealth Reports (1997-2015). We compute the PRIX for historical data and show that its past behavior is in line with the expected features of a risk index. Section 4 illustrates a potential application of the PRIX, and Section 5 concludes.

2. Desirable features of the *P*rivate investor *R*isk *I*nde*X* and shortcomings of classical risk measures

The main idea behind the creation of a benchmark risk index such as the PRIX is to provide private investors with useful information about risk, aggregated both across national borders and asset classes. This will be achieved by tracking the risk of the PRIP using an appropriate risk metric. We think that desirable features of the PRIX include the following: it should provide precise, straightforward and easy-to-understand information, be always available (at least at a daily frequency) and be transparent, i.e. it should be based only on publicly available information (as opposed to proprietary data). It should serve as a benchmark against which private investors can easily compare the risk of their own portfolios[4]. In the following, we review existing metrics of risk (including classical risk measures) focusing on the information they process and the features they provide. This discussion will then lead us to develop our own risk metric (PFT) to be used as the basis for the PRIX.

For reasons of availability and transparency, we want to base the PRIX solely on returns series, which are widely available on a daily basis. Given this decision, there are a number of alternative risk metrics which require only information on the portfolio's return series, for example, classical risk measures such as volatility and (conditional) value-at-risk. As volatility is not directly observable, it must be calculated based on some model. Historical measures of volatility, whether based on simple or exponential moving averages (JP Morgan, 1996) of past squared returns, provide a trade-off between a focus on long-term risk characteristics and the current level of risk. Conditional (current) volatility measures, like those based on the well-known (G)ARCH-models of Engle (1982) and Bollerslev (1986), need to be properly specified and are subject to additional sources of uncertainty such as estimation error. This can have a serious impact on the resulting volatility measure and thus might reduce the usefulness of such models in the context of a risk index. Initially, implied volatility indices such as the VIX[5] were also subject to specific modeling assumptions. Since the seminal work of Britten-Jones and Neuberger (2000), implied volatility indices no longer depend on any particular option pricing model. However, they still require highly liquid option markets, which do not exist for most asset classes included in the PRIP. In contrast to this, we want to base our risk index on a metric that works for all asset classes, also for those without a liquid option market. Additionally, it should be sensitive to multidimensional aspects of risk such as unusualness in return correlations.

Other risk metrics, such as coherent risk measures in the sense of Artzner *et al.* (1999), suffer from similar problems, as they either require proper specification and depend on volatility with all its restrictions mentioned above[6] or have to be sampled from historical time series and thus lack the focus on current levels of risk we desire for the PRIP. Similar to volatility and value-at-risk, which are commonly used in portfolio and risk management, the PFT risk metric derived in this paper is not coherent in the sense of Artzner *et al.* (1999); yet, it shows many desirable characteristics described at the beginning of this section.

Apart from the risk *within* individual asset classes, the *dependence* between these asset classes is also an important driver of total risk in the PRIP. The most widely used measure for this source of risk is correlation, which captures linear dependence. Similar to volatility, estimating correlations from data[7] is based on a trade-off between long-term characteristics and the current (conditional) correlation. Multivariate time-series models for the entire covariance matrix, such as multivariate GARCH (Engle, 2002; Lütkepohl, 2006) also suffer from problems associated with proper specification and estimation errors. The same is true for other time-series-dependent models of correlation, such as asymmetric or regime-shifting correlations (Ang and Chen, 2002; Ang and Bekaert, 2002). Implied correlation indices such as the JCJ[8] require liquid option markets, which do not exist for most asset classes in the PRIP. Finally, we highlight that although changes in correlation affect portfolio volatility, correlation risk or deviations of correlations from their usual levels are not separately taken into account when using portfolio volatility as a risk metric. For this reason, we do not use portfolio volatility as the risk metric for constructing the PRIX, but we derive instead an appropriate risk metric that overcomes these restrictions.

Based on this review of the best-known metrics and measures of risk, we summarize the requirements for such a risk metric. First, as derivative securities are unavailable for some of the asset classes in the PRIP, our risk metric should be based solely on the returns of the relevant time series. Second, because we want the PRIX to be a measure of current (conditional) risk in the market signaling, where the risk level is currently, it must be unaffected by the past (unlike, for example, moving averages of correlation and volatility). In addition, as we find our data to be non-normal, the metric should be free from strong distributional assumptions. It should be scale-independent to allow for easy comparison of an investor's portfolio risk to the risk of the benchmark portfolio independent of portfolio size, and it should allow for risk attribution across constituent asset classes. Finally, it should take dependencies between these asset classes as well as changes within these dependencies properly into account. In the next section, we will derive such a risk metric and construct our risk index PRIX by applying this metric to the PRIP.

3. A benchmark risk index for global private investors

3.1 P*ort*F*olio* T*urbulence*

A number of properties discussed in the previous section are satisfied by the FT metric originally introduced by Chow *et al.* (1999) and Kritzman and Li (2010). FT is sometimes also referred to as "multidimensional z-score" or "Mahalanobis distance" because of Mahalanobis (1927, 1936). Previous applications of this metric in finance include, for example, Meucci (2009) for asset allocation purposes and Geyer *et al.* (2014) for the generation of arbitrage-free financial scenarios. Lütkepohl (2006) uses the Mahalanobis distance in the construction of forecasting regions in the form of ellipsoids[9].

FT measures the "unusualness" of financial asset returns with regard to some reference level, which is commonly set to their historical pattern of behavior. Unusualness in this regard includes extreme price movements, decoupling of correlated assets or convergence of previously uncorrelated assets (Kritzman and Li, 2010). Such unusualness in the markets (as measured by FT) has been used by Berger (2013) to estimate market betas that are quite consistent with predictions from the Capital Asset Pricing Model (CAPM). Also, Giglio *et al.* (2016) used FT as best predictor of real economic activity measured by the Chicago Federal Reserve (FED) national activity index.

FT on day t is defined as:

$$FT_t = \sqrt{(r_t - \mu)' \Sigma^{-1} (r_t - \mu)} \tag{1}$$

where r_t is a vector of returns on day t, μ a vector of reference (historical) means and Σ the reference (historical) covariance matrix. Thus, FT measures not only deviations of current returns from historical means but also deviations from historical correlations. However, because FT does not take portfolio weights into account, all component assets implicitly receive identical weights and, hence, contribute equally to its value.

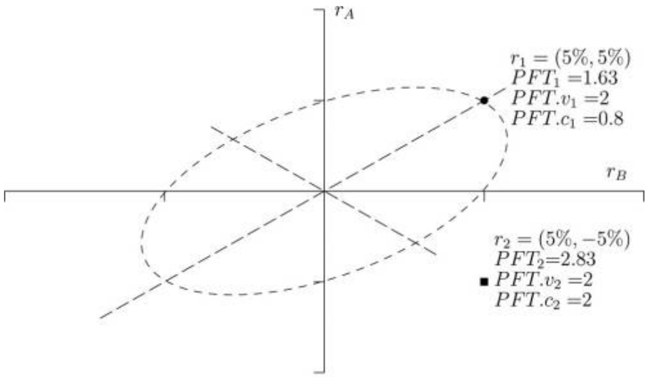
PRIX is intended to provide a suitable benchmark, which allows private investors to relate the level of risk in their respective portfolios to the benchmark portfolio. To arrive at a risk metric featuring the required scale-independence, properly accounting for portfolio weights and allowing for a comparison of portfolios across arbitrary sets of financial assets, we modify FT. The desired changes can be achieved by incorporating a diagonal matrix of (possibly time-varying) weights $w_{t,D}$ into the original FT metric as given in equation (1). A similar approach of using portfolio weights as an input in a total portfolio context has been used by Rudin and Morgan (2006), however, with a different application, namely, to calculate a portfolio diversification index. Keeping in mind that the mean μ_t and the covariance matrix Σ_t may be time-varying (e.g. computed from a n-day historical rolling window from $t-n$ to $t-1$), we define PFT_t (based on the weighted Mahalanobis distance) as:

$$\mathrm{PFT}(r_t; w_t, \mu_t, \Sigma_t) := \sqrt{\frac{1}{\sum_{i=1}^{n} w_{t,i}^2}(w_{t,D}(r_t - \mu_t))' \, \Sigma_t^{-1}(w_{t,D}(r_t - \mu_t))} \tag{2}$$

The scaling factor equals the expected value implied by the weighting of the elements in the Mahalanobis distance[10].

Figure 1 illustrates this risk metric for the simple example of two correlated assets A and B [based on Kinlaw and Turkington (2013)]. Assuming historical means of 0, volatilities of 0.05 and a correlation of 0.5, the equally weighted PFT of the first pair of returns $r_1 =$ (0.05,0.05) is 1.63. The return combination r_1 is depicted as a solid point, and the ellipse shows all return combinations that exhibit the same level of PFT. Its elliptical shape is determined by the covariance matrix Σ. We notice that the returns are exactly one standard deviation from their historical means and deviate from 0 in the same direction as they would for a correlation of 1. The second return combination is depicted as the solid square for $r_2 =$ (0.05, -0.05). While these returns are also exactly one standard deviation from their historical means, they deviate in different directions, as they should for a perfectly negative correlation. Compared to their historical correlation of 0.5, the second return pattern is more unusual than the first one. This is reflected by a higher (equally weighted) PFT value of 2.83 for the second set of returns. For the equally weighted portfolio, the first return is 0.05, whereas the second return is equal to zero (its historical mean). While portfolio volatility (in terms of weighted squared deviations from μ) on both days is identical, PFT_2 is substantially higher because it reflects unusualness not only in the level of the returns but also in the direction of moves. This is the reason why it is located far outside the ellipse determined by the first combination of returns, which corresponds to a PFT value of 1.63. This illustrates the advantage of PFT over portfolio volatility. If we additionally let the weight of any asset go to zero, PFT in both cases would be equal to one, highlighting the fact that realized returns are exactly one standard deviation away from their respective mean.

The deviation from reference values as measured by PFT can be split up into a part that comes from unusualness in volatility (PFT.v)[11] and a part that comes from unusualness in



Notes: In this figure, we depict the location of two return combinations r_1 and r_2 of assets A and B. The location of r_1 is denoted by a solid point, the ellipsis depicts the location of all return combinations that would exhibit the same PFT; the location of r_2 is shown by a solid square

Figure 1.
Iso-risk ellipse for different return combinations

correlation (PFT.*c*). Such a decomposition has been suggested by Kinlaw and Turkington (2013) for FT. It can be modified for application to PFT as follows (also depicted in Figure 1):

$$\mathrm{PFT}_t\,.\,v{:}=\sqrt{\frac{1}{\sum_{i=1}^{n} w_{t,i}^2}(w_{t,D}\;(r_t-\mu_t))'\,\Sigma_{t,D}^{-1}(w_{t,D}(r_t-\mu_t))} \tag{3}$$

$$\Sigma_{t,D}{:}=\begin{pmatrix}\sigma_{t,1}^2 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \sigma_{t,n}^2\end{pmatrix} \tag{4}$$

$$\mathrm{PFT}_t\,.\,c{:}=\frac{\mathrm{PFT}_t}{\mathrm{PFT}_t\,.\,v} \tag{5}$$

Table I compares PFT to FT and classical risk measures and indicates whether or not they fulfill the desirable criteria discussed above.

3.2 A representative (benchmark) portfolio for internationally diversified private investors

In this section, we define the benchmark portfolio PRIP as the average portfolio composition of an internationally diversified private investor. To this end, we use the World Wealth Reports of Cap Gemini (1997-2015), which provide information and data on the wealth of HNWIs for 1996-2014. In 2012, roughly 12 million HNWIs (each holding, by definition, more than US$1m in financial assets) owned US$46.2tn in financial assets. Comparing this to the size of the Global (multi-asset) Market Portfolio of Doeswijk *et al.* (2014), which has a total market value of US$90.6tn at the end of 2012, we find that HNWIs own roughly half of the world's financial assets. Although not all globally active private investors are HNWIs, these figures justify using the average portfolio composition of HNWIs as the representative (benchmark) PRIP. Figure 2 provides an overview of the number of HNWIs from 2009 to 2014, as well as their holdings of financial assets.

Risk measure/metric	Calculates current risk	Requires no model	Considers asset-class interdependency	Includes portfolio weights	Is available for all asset classes
Volatility					
(W.) Moving Average		(x)		x	x
Model-based (GARCH)	x			x	x
Implied	x	x			
Correlation					
(W.) Moving Average			x	x	x
Model-based (GARCH)	x	x	x		
Implied					
Unusualness					
Financial turbulence	x	x	x		x
Portfolio turbulence	x	x	x	x	x

Note: It indicates whether or not they fulfill the criteria discussed in the text as desirable for our multi-asset risk index

Table I. This table compares different risk measures discussed in Section 2 to the risk metrics FT and PFT as defined in 3

Private investors predominantly hold assets belonging to the following five asset classes: equities, fixed income, real estate, cash/deposits and alternative investments[12]. Table II displays the average asset allocation of HNWIs from 1997 to 2014.

Based on the average asset allocation of HNWIs in Table II, we choose indices to represent these asset classes and calculate their weights in the PRIP. To represent global equity and global fixed income, we choose the *FTSE ALL WORLD* index and the *Barclays Multiverse All* index, as both of them are the broadest indices available for their respective asset classes with a sufficiently long history. Table III shows that they represent almost 100 per cent (equity: US$30.83 of 32.92tn, fixed income: US$45.02 of 49.77tn) of their respective share in the Global Market Portfolio of Doeswijk *et al.* (2014). Compared to the share of real estate in this Global Market Portfolio, the index coverage of global real estate, represented by the *FTSE EPRA/NAREIT Global* index, is quite small. Note that the share of real estate in the PRIP (between 15 and 20 per cent) is larger than in the Global Market Portfolio of Doeswijk *et al.* (2014), which is supported by evidence provided by the real estate industry[13]. As we seek to evaluate only the risk of the risky portfolio in a two-fund-separation sense, independent of any notion of risk aversion, we drop cash and deposits, which we consider as risk-free assets in the short run.

Finally, we proxy alternative investments by commodities only, represented by the *S&P GSCI* commodity index. This is because of the lack of representative index data for other sub-classes of alternative investments, such as hedge funds, currencies, venture funds or fine arts and

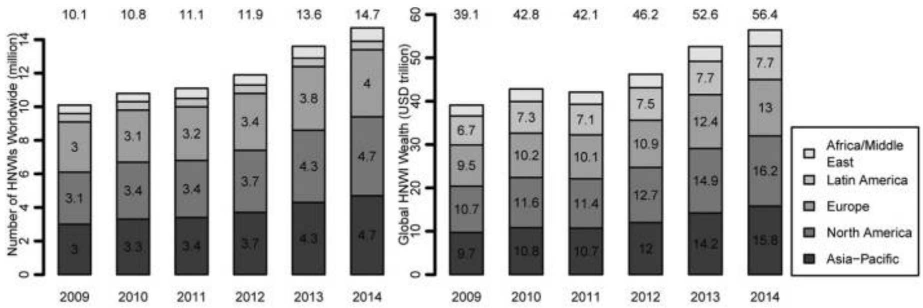


Figure 2.
Number (million) and aggregated wealth (USD trillion) of HNWIs worldwide, development 2009-2014

Source: Cap Gemini World Wealth Reports 1997-2015

Table II.
HNWI wealth allocation to different asset classes (selected periods 1997-2014)

Year	1997 (%)	2002 (%)	2003 (%)	2004 (%)	2005 (%)	2007 (%)	2008 (%)	2011 (%)	2012 (%)	2013 (%)	2014 (%)
Equities	34	20	24	28	30	33	25	30	26	26	25
Fixed income	26	30	27	24	21	27	29	22	16	16	16
Real estate[a]	17	15	16	16	16	14	18	20	20	20	19
Cash/deposits	20	25	19	13	13	17	21	21	28	28	27
Alternative Investments[b]	4	10	15	19	20	9	7	8	10	10	14
Total	100	100	100	100	100	100	100	100	100	100	100

Notes: [a]Includes commercial real estate, REITs, residential real estate (excluding primary residence), undeveloped property, farmland and other; [b]Includes structured products, hedge funds, derivatives, foreign currency, commodities, private equity, venture capital and investments of passion (fine art and collectibles)
Source: Cap Gemini World Wealth Reports 1997-2015

Asset class	Equities	Fixed income	Real estate	Commodities[a]	Portfolio
Data	FTSE ALL WORLD	Barclays multiverse all	FTSE EPRA/NAREIT Global	S&P GSCI Commodity	*Weighted average*
Start date	31.12.1993	01.09.2000[b]	31.10.2008[c]	01.01.1970	*01.01.1994*
Index Mkt. Val.[d]	30.83	45.02	1.15		
Global Mkt. PF[e]	32.92	49.77	4.61		*90.57*[f]
PRIP[g]	11.98	7.32	9.31	4.66	*33.26*[h]

Notes: [a] Alternative Investments only represented through commodities which in 2010 represented 22% of AI (Foreign Currency 15%, Hedge Funds 24%). Source: (Cap Gemini, 2011); [b] Daily data filled back to 01.01.1999 with Barclays Global Aggregate Index (represents 96% of the Barclays Multiverse All Index), and to 01.01.1994 with Barclays US Aggregate (represents 51% of the Barclays Multiverse All Index); [c] Daily data filled back to 01.01.1994 with FTSE EPRA/NAREIT Developed Index (represents 95% of the FTSE EPRA/NAREIT Global Index); [d] Market value of the respective asset class index at 31.12.2012 (USD trillion); [e] Market value of Global (investable) Market Portfolio (USD trillion) at the end of 2012, according to Doeswijk et al. (2014); [f] In the Global (investable) Market Portfolio of Doeswijk *et al.* (2014) cash/deposits are not taken into account; [g] Market value of the PRIP per asset class at 31.12.2012 (USD trillion), for example, equities = $46.2·(1 − 0.28)·0.36 = $11.98tn; [h] As cash does not add any substantial risk to the portfolio and would therefore have no influence on the risk measure we omit this asset class (28% of US$46.2tn)
Source: Thomson Reuters Datastream

Table III. Description of time series used for representation of PRIP asset classes

collectibles. In 2010, commodities constituted roughly 22 per cent of alternative investments (Cap Gemini, 2011), so this choice may lead to some differences between the PRIP and the average market portfolio of HNWIs. Figure 3 shows the final portfolio weights of the four asset classes in the PRIP (right) in comparison to the historical HNWI wealth allocation.

Table IV shows descriptive statistics of the time series used to represent the asset classes in the PRIP, together with the statistics for the aggregate portfolio. Equity and real estate yielded the highest average returns while, at the same time, exhibiting also high volatility. In contrast, commodities exhibited the lowest average return but the highest volatility of all asset classes. We also observe negative skewness for equities, real estate and commodities but slightly positive skewness for fixed income. Excess kurtosis is positive for all asset classes and very large for real estate with 10.53 (other asset classes between 3.25 and 8.04). A quick check on normality of the return series rejects the normality assumption for all asset classes at the 99.99 per cent level. Checking for dependencies between asset classes, we note that real estate is highly correlated with equities ($\rho = 0.75$), whereas all other correlations are markedly lower, ranging between 0.06 and 0.30.

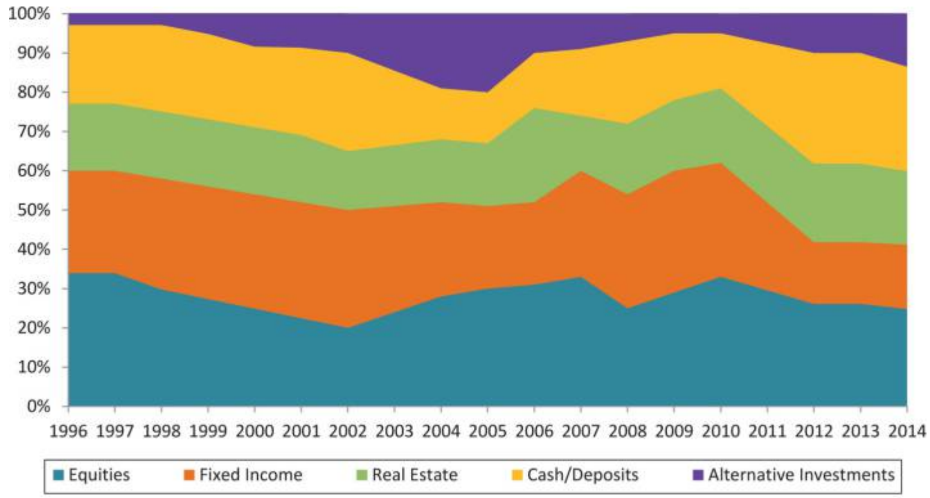
Concluding this section, we want to highlight the most important characteristics of the PRIP. It represents the average portfolio of HNWIs worldwide. Weights for asset classes in the PRIP are different from those in the global market portfolio because private investors and institutional investors have different asset allocations. Constructed as a global average, the PRIP does not suffer from any home bias. In the next subsection, we describe the derivation of the PRIX in detail.

3.3 Definition and properties of the Private *investor* Risk IndeX

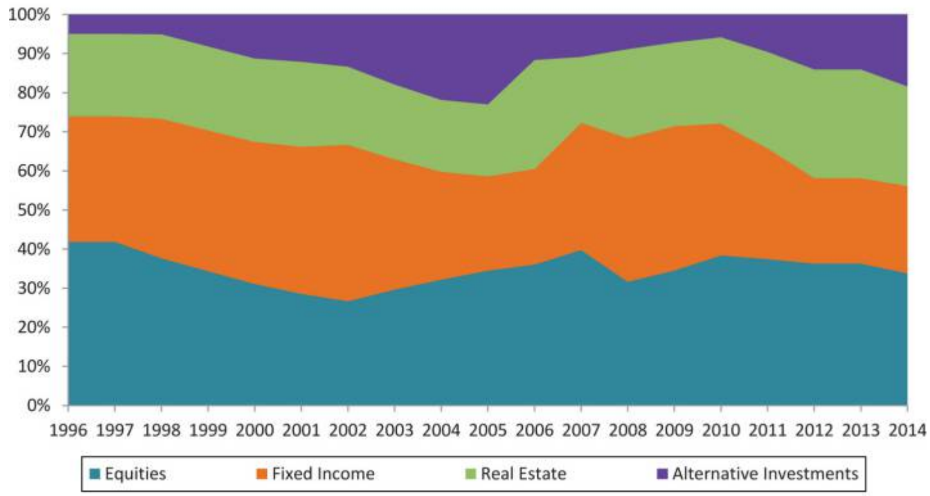
We define the PRIX by applying PFT to the PRIP as derived in Section 3.2 as:

$$\text{PRIX}_t := \text{PFT}(r_{\text{PRIP},t}; w_{\text{PRIP},t}, \mu_{\text{PRIP},t}, \Sigma_{\text{PRIP},t}) \quad (6)$$

where $\mu_{\text{PRIP},t}$ and $\Sigma_{\text{PRIP},t}$ are computed from a recursive window ending at time $t-1$, and $r_{\text{PRIP},t}$ and $w_{\text{PRIP},t}$ are the current returns and weights of the PRIP. Using a recursive approach is



(a)



(b)

Figure 3.
(a): HNWI wealth allocation to different asset classes (selected periods 1997-2014). (b): Portfolio weights in the PRIP (own calculations)

based on the assumption of ergodicity of the return processes for the asset classes in the PRIP, which implies convergence to the long-term values μ_{PRIP} and Σ_{PRIP} for increasing t.

To get some feeling for the behavior of the PRIX, we calculate it for historical data and compare it to existing indices commonly taken as indicators of risk and market fear (e.g. the VIX) (Whaley, 2000). We would expect the PRIX to show the following characteristics:

- clear reaction to well-known events of financial market turmoil;
- persistence in time (Kritzman and Li, 2010)[14]; and
- usefulness in predicting future risk and returns-to-risk (Whaley, 2009).

In the upper part of Figure 4, we calculate the PRIX and plot it against the total return of the PRIP. One can easily observe that spikes of the PRIX are related to well-known incidents of financial market turmoil and marked losses of the PRIP. In this calculation, we follow Bloom (2009) and take an ex-post view on market unusualness by calculating the PRIX based on μ_{PRIP} and Σ_{PRIP} estimated from the full sample of PRIP returns. Hence, in contrast to later applications, we deliberately take all information into account when assessing market unusualness, including information that became available only after the respective events occurred. In retrospect, we find the highest levels of financial market turmoil during the financial crisis (from 2007-2010). In these years, real estate, fixed income and equity markets faced sizeable losses simultaneously, exhibiting high levels of unusualness in returns and correlations. Other periods of significant unusualness in markets include the Asian crisis in 1997, the collapse of LTCM, the burst of the technology bubble, 9/11, the defaults of Enron and WorldCom, the "flash crash" in 2010, the US rating downgrade, the 2013 "mini crash" and the 2015 Chinese "Black Monday". To illustrate different behavior in turbulent vs non-turbulent periods, Table V shows descriptive statistics of the PRIP for the complete data set as well as for corresponding sub-samples. Turbulent periods are determined for the top decile of days in our sample. We find the average PRIX level from 1994-2015 to be 0.86, whereas in turbulent periods (PRIX levels between 1.56 [90 per cent-quantile] and almost 6.71), we have an average PRIX-level of 2.14 (as opposed to 0.71 in non-turbulent periods).

During financial market turmoil, the portfolio return drops significantly below zero (from 5.85 to −26.78 per cent, all statistics annualized), whereas volatility rises from below 10 to nearly 22 per cent. Broken down by asset classes, the second column shows that average returns on turbulent days are negative for three out of four asset classes. The only exception to this is fixed income, whose average returns on turbulent days are more than twice as high as on non-turbulent days. Standard deviations are markedly higher on turbulent days for all asset classes.

Further analysis reveals a qualitative change in correlations between asset classes in turbulent times. Before the financial crisis that started in 2007, markets exhibited a so-called "flight-to-quality" behavior, where in turbulent periods investors turned away from equities and shifted their funds into cash and fixed income securities. This caused a negative correlation between equities and fixed income as opposed to the low positive correlations we observe under normal market conditions. In the financial crisis, however, correlations suddenly increased markedly even for asset classes that previously showed negative or low positive correlations. This is one of the reasons for the high PRIX values during the financial crisis.

As mentioned in the introduction to this section, another well-known feature of other risk measures is their persistence through time (e.g. "volatility clustering"). Table VI provides information about the persistence and predictive power of the PRIX. As the persistence and implied return predictability (discussed below) of a risk index are important features from the perspective of an investor holding the PRIP, we now take an *ex-ante* perspective and

Global asset class	Mean	SD	Skewness	Excess Kurtosis	Correlation				JB-Test
Equities	6.72	15.03	−0.38	8.04	1.00	0.06	0.75	0.30	15,455***
Fixed income	4.65	5.02	0.06	3.51		1.00	0.09	0.07	2,924***
Real estate	7.24	15.99	−0.52	10.53			1.00	0.24	26,530***
Commodities	1.16	21.45	−0.25	3.25				1.00	2,569***
Total portfolio	5.85	9.47	−0.56	8.53					17,539***

Notes: We depict statistical significance at the 1%, 0.1% and 0.01% level by *, ** and ***, respectively
Source: Thomson Reuters Datastream

Table IV. Annualized descriptive statistics and Jarque–Bera tests on normality (test statistics and level of significance) of asset class time series in the PRIP (01.01.1994-08.10.2015, n = 5,679)

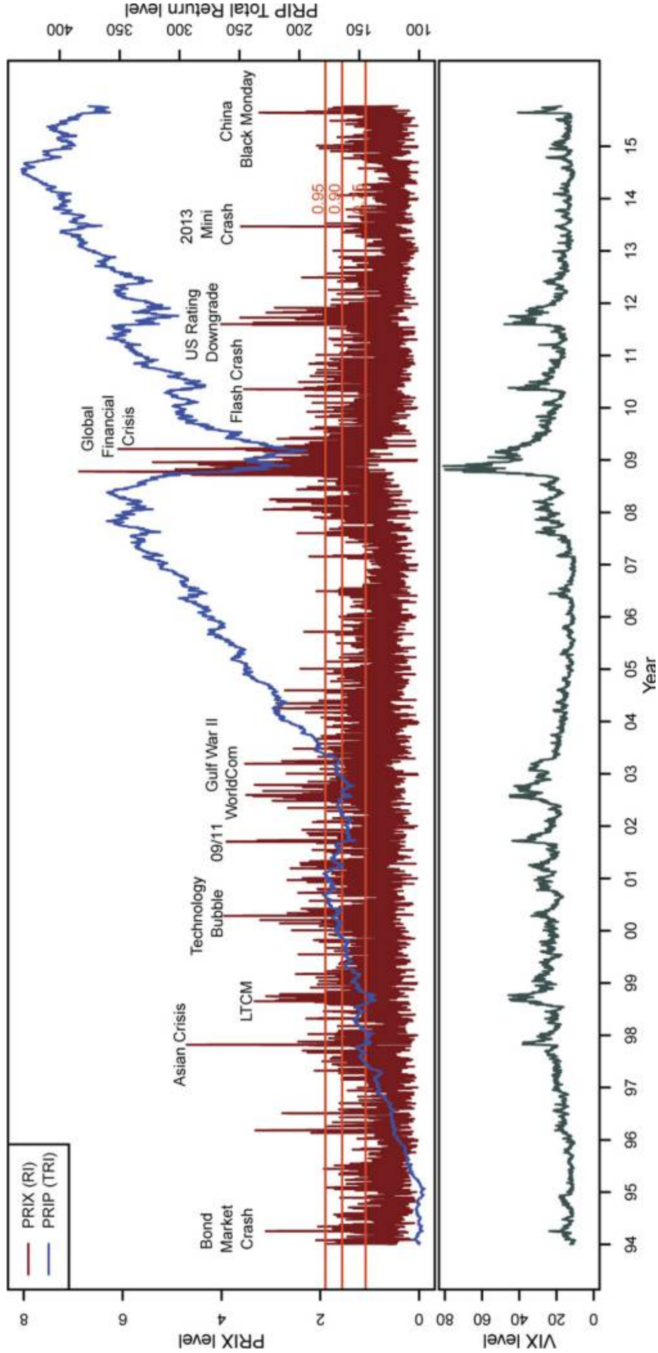


Figure 4.
PRIX and PRIP Total Return Index (upper plot) in relation to VIX (lower plot) 1994-2015, selected market events (computed from entire dataset following Bloom (2009))

Global asset class	Mean	SD	Skewness	Excess Kurtosis	Correlation			
Entire dataset								
Equities	6.72	15.03	−0.38	8.04	1.00	0.06	0.75	0.30
Fixed income	4.65	5.02	0.06	3.51		1.00	0.09	0.07
Real estate	7.24	15.99	−0.52	10.53			1.00	0.24
Commodities	1.16	21.45	−0.25	3.25				1.00
Total portfolio	5.85	9.47	−0.56	8.53				
Turbulent days[a]								
Equities	−37.20	36.00	−0.03	0.11	1.00	0.04	0.81	0.48
Fixed income	9.88	9.93	0.02	0.33		1.00	0.09	0.10
Real estate	−49.89	36.31	−0.13	1.52			1.00	0.43
Commodities	−62.93	37.11	−0.21	1.23				1.00
Total portfolio	−26.78	21.89	−0.18	0.62				
Non-turbulent days								
Equities	11.60	10.31	−0.17	0.32	1.00	0.09	0.68	0.17
Fixed income	4.06	4.13	−0.04	0.37		1.00	0.10	0.06
Real estate	13.59	11.66	−0.15	1.74			1.00	0.12
Commodities	8.28	18.88	−0.03	1.30				1.00
Total portfolio	9.48	6.78	−0.07	0.6				

Notes: [a] Turbulent days are those where the PRIX is higher than 1.56, which corresponds to the 90%-quantile. Average levels of the PRIX on the entire dataset, turbulent and non-turbulent days are, respectively: 0.86, 2.14 and 0.71. The PRIX values were computed from the entire dataset (using an ex-post approach) following Bloom (2009)
Source: Thomson Reuters Datastream and own calculations

Table V. Annualized descriptive statistics for the PRIP (asset classes and total portfolio) for the entire dataset (n = 5,679), turbulent and non-turbulent days (annualized data)

calculate the PRIX of day t based on μ_t and Σ_t derived from the historical set of returns up to $t-1$ (i.e. in contrast to the previous example, we use only information that was available at the time of computing the index). The first section depicts average levels of the PRIX, PRIP returns and volatilities for the days, where the PRIX exceeds the 75, 90 or 95 per cent-quantile of PRIX levels in the recursively growing sample. The next three sections show the average levels of the PRIX, PRIP returns and PRIP volatilities for the 5, 10 and 20 days, respectively, after the initial threshold-crossing of the PRIX. In the last section, we depict the full sample PRIX, PRIP returns and PRIP volatility as a benchmark.

The second column of Table VI shows that once the PRIX crosses a certain (high) threshold level, it remains high (quantile in relation to the growing sample depicted in parentheses) for at least 20 days. For example, the average level of the PRIX after exceeding the 90 per cent-quantile is 2.06, and, on the five days after this event, we still see an average PRIX level of 1.07, which corresponds to the 75th percentile. For both the average across 10 and 20 days after exceeding the 90 per cent-quantile, we get 1.06, which corresponds to the 74th percentile and clearly shows the persistence of the PRIX. Another important feature of risk indices (apart from their time persistence) is their ability to predict future risk and returns. While the predictability of future risk is closely related to time persistence, the predictability of returns is no straightforward feature. It is related to the fact that after the initial breakdown of returns and the initiation of a period of higher volatility, returns should increase to compensate for the now increased level of risk in the markets. We find elevated levels of returns in Table VI for the next 5, 10 and 20 days after the respective 75, 90 and 95 per cent-quantile exceedance. In addition, we find higher return-to-PRIX performance (except for the 75 per cent next five days) compared to the entire sample. In conclusion, we showed that similar to well-known risk indices, such as the VIX, the PRIX shows the expected behavior of

Days	PRIX *(quantile)*	PRIP Return	PRIP SD	PRIP Return/PRIX
Day of reading				
75	1.53	−0.06	0.13	
90	2.06	−0.29	0.20	
95	2.46	−0.50	0.25	
Next 5 days				
75	0.93 *(66)*	0.06	0.10	0.06
90	1.07 *(75)*	0.10	0.12	0.09
95	1.28 *(81)*	0.04	0.16	0.03
Next 10 days				
75	0.93 *(66)*	0.07	0.10	0.07
90	1.06 *(74)*	0.08	0.12	0.07
95	1.25 *(82)*	0.06	0.15	0.05
Next 20 days				
75	0.93 *(66)*	0.06	0.10	0.07
90	1.06 *(74)*	0.06	0.12	0.06
95	1.23 *(80)*	0.02	0.15	0.02
Full sample	0.86	0.06	0.09	0.07

Note: The last line shows the full sample average PRIX, returns and volatilities to simplify comparisons
Source: Thomson Reuters datastream and own calculation

Table VI. Level of the PRIX, annualized return and risk and return-to-PRIX-ratios on the initial reading of each sequence of 75%, 90% and 95% quantile events and the 5, 10 and 20 days after the reading

reacting to financial market turmoil, being time-persistent and implying predictability of future risk and returns-to-risk.

4. Application for tactical asset allocation

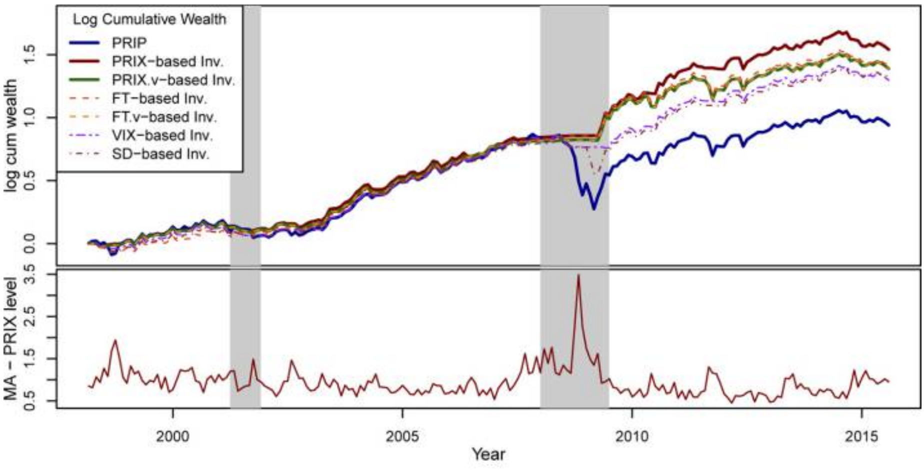
To show one potential application of the PRIX, we follow Berger and Pukthuanthong (2016) and use the PRIX as the basis for a simple but effective tactical asset allocation strategy. At the beginning of each month, a hypothetical global private investor decides whether or not to go long in the PRIP. His decision is based on the level of the 20-day moving average (MA) of the PRIX in relation to its historical (recursive) 75 per cent-quantile[15]. We compare this simple strategy to similar in-and-out strategies based on the levels of a 20-day MA of PRIX.v (to highlight the impact of unusualness in correlation), FT, FT.v, the VIX and the 20-day historical standard deviation. We take turnover into account and assume transaction costs of 50 bp per trade. As global risk-free rate, we take the one provided by Kenneth French on his homepage (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html) calculated as described in Fama and French (2012).

We find the investment strategy using PRIX to outperform all other strategies in terms of Sharpe ratio, Sortino ratio and Return-Loss calculations. As can be seen in Figure 5 and Table VII, the asset allocation strategy based on the PRIX produces the highest level of average returns (0.097) together with a low average value of standard deviation (0.079). In terms of risk-adjusted performance, the Sharpe and Sortino ratios show that the PRIX-based strategy dominates all other investment strategies while, at the same time, having less turnover than the strategy based on historical standard deviation. To account for the impact of transaction costs (assumed at 50 basis points), we follow De Miguel *et al.* (2009) and calculate the return-loss against the buy-and-hold-strategy. Interpreting the return-loss of the PRIX compared to the buy-and-hold strategy, we find that the returns of the PRIX strategy could be 4.6 per cent lower and would still exhibit the same Sharpe ratio. All other return-loss factors are considerably lower. We also find that an exclusion of unusualness in correlations (from PRIX to PRIX.v) yields a lower performance and that the transformation from FT to PFT (PRIX) does enhance the performance significantly. The strategy outperforms in spite of its higher turnover and despite accounting for transaction costs of 50 bp per trade. As one of the main features of the PRIX is that it measures unusualness in returns, it is not surprising to find the main outperformance of the PRIX-based investment strategy during the financial crisis. In more "normal" market conditions, its performance is closer to the other strategies.

Panel B of Table VII depicts p-values of Memmel (2003) tests on the Sharpe ratios of the row-strategies being larger than those of the column-strategies. In the second line, we find that the null hypothesis that the Sharpe ratios of the other strategies are higher than the one from the PRIX-based strategy can be rejected for standard significance levels, except for the VIX, where we can only reject for a level of 14 per cent. While all the other strategies also outperform the buy-and-hold strategy, none is able to beat the PRIX. Figure 5 displays a plot of the logarithmic cumulative wealth of all strategies including rebalancing costs and corrected turnover calculation according to De Miguel *et al.* (2009). One can easily see that the PRIX-based strategy outperforms others in turbulent times, especially during the financial crisis and hardly ever underperforms. Again, we find that our transformation of FT to PFT does add significant improvements not only in Sharpe ratios but also in cumulative wealth.

5. Conclusion

In this paper, we constructed a risk index for private investors (the PRIX). We started by arguing that PFT, which we developed as a weighted version of FT, is a risk metric that is well-suited for constructing such a risk index. We then investigated the average investment portfolio for a globally active private investor (the PRIP) and derived the PRIX by applying PFT to the PRIP. We found that the PRIX not only overcomes many of the problems of traditional risk indices, such as the requirement for the existence of a liquid option market, but also incorporates additional information on unusualness in inter-asset dependence. This has been illustrated using numerical examples. The PRIX



Note: The grey areas represent times of US-recession (NBER-shades)

Figure 5. Log Cumulative Wealth including rebalancing cost (50 bp) of investments in the PRIP (03/1998-10/2015) with regard to a buy-and-hold-strategy (blue line) and investments based on a monthly in-and-out-strategy with regard to a 20-day MA of the PRIX (red line), PRIX.v (green line), FT (orange line), FT.v (yellow line), VIX (purple line) and a 20-day historical standard deviation (brown line)

Investment Strategy	Mean return	SD	Sharpe ratio	Sortino ratio	Turnover	Return-loss	Total Return
Panel A							
Buy-and-hold	0.054	0.124	0.27	0.40			11.29
PRIX	0.097	0.079	0.96	1.98	78.6	−0.046	20.27
PRIX.v	0.087	0.088	0.75	1.34	40.0	−0.035	18.18
FT	0.090	0.088	0.78	1.40	55.5	−0.036	18.91
FT.v	0.087	0.088	0.75	1.34	40.0	−0.035	18.18
VIX	0.081	0.077	0.79	1.47	46.8	−0.033	17.07
Hist. SD	0.086	0.089	0.74	1.26	87.5	−0.031	18.16
	Buy-and-Hold	PRIX	PRIX.v	FT	FT.v	VIX	Hist. SD
Panel B							
Buy-and-Hold	NA	1.00	0.99	0.99	0.99	0.98	0.97
PRIX	0.00	NA	0.06	0.09	0.06	0.14	0.03
PRIX.v	0.01	0.94	NA	0.50	1.00	0.57	0.35
FT	0.01	0.91	0.50	NA	0.50	0.56	0.35
FT.v	0.01	0.94	1.00	0.50	NA	0.57	0.35
VIX	0.02	0.86	0.43	0.44	0.43	NA	0.30
Hist. SD	0.03	0.97	0.65	0.65	0.65	0.70	NA

Notes: Panel A shows annualized monthly mean, standard deviation, Sharpe and Sortino ratio of a buy-and-hold investment in the PRIP (03/1998-10/2015) and investments based on a monthly in-and-out-strategy with regard to a 20-day moving average of the PRIX, PRIX.v, FT, FT.v, VIX and a 20-day historical standard deviation. The last two columns show the portfolio turnover based on the respective strategy and the return-loss following De Miguel *et al.* (2009) assuming transaction cost of 50BP. The global one month risk-free rate comes from Kenneth French's data library (URL is provided in the text) and is used for out-of-PRIP-investments and to determine the Sharpe and Sortino ratios; Panel B reports the p-values of Memmel (2003) tests on the Sharpe ratios of the row-strategies being larger than those of the column-strategies. Sharpe ratios were calculated including transaction cost
Source: Thomson Reuters Datastream and own calculations

Table VII.
Portfolio statistics for risk based asset allocation strategies applied to the PRIP (03/1998-10/2015)

also shows the behavior usually expected of a risk index when calculated for historical data: it rises during known historical periods of market turmoil, and it shows persistence and forecasting power for the prediction of risk and returns conditionally on heightened levels of the PRIX for up to 20 days ahead. A dynamic in-and-out asset allocation strategy was used to indicate a possible application of the PRIX and illustrated its outperformance in relation to other well-known indices of risk, such as the VIX, historical standard deviation or (unweighted) FT. An interesting question for further research is whether risk premia in the component asset classes can be forecast based on the PRIX. Combined with the persistence in the PRIX demonstrated in this paper, implying forecasting power for the amount of risk, this could provide a basis for profitable trading strategies.

Notes

1. This will be illustrated in Section 3.2.
2. A survey of such risk indices has been carried out by the US Department of Treasury (Bisias *et al.*, 2012), the European Central Bank (ECB) (2010), De Bandt and Hartmann (2000), the International Monetary Fund (2009) and others (Billio *et al.*, 2012; Acharya *et al.*, 2010).
3. We are aware that Berger and Pukthuanthong (2016) develop an index of market fragility and stress, explicitly calling it "a novel risk measure". However, we refrain from calling the metric behind our risk index *measure* for the reasons stated above.
4. Therefore, the risk metric used to construct the index should be scale-independent, i.e. its magnitude should not depend on the size of the portfolio.

5. VIX is based on S&P 500-options, other implied volatility indices are based on US (VXN: NASDAQ, VXD: DJIA) and European Stocks (VSTOXX: EURO STOXX 50, VDAX: DAX30).
6. Examples for coherent risk measures that require distributional assumptions are Conditional Value-at-Risk (Rockafellar and Uryasev, 2002; Uryasev, 2004), Entropic Value-at-Risk (Ahmadi-Javid, 2011) or Superhedging Prices (Follmer and Schied, 2010).
7. Similar to volatility, correlation, apart from the standard measurement as simple (moving) average, can be estimated as exponential moving average (JP Morgan, 1996).
8. Similar to the VIX, the CBOE provides correlation indices on the S&P 500, such as the JCJ, KCJ and ICJ for maturities of January 2017, 2018 and 2019.
9. For a detailed literature review, a categorization according to the type of input parameters and some examples see Stöckl and Hanke (2014).
10. It can be shown that the (squared) Mahalanobis Distance is the sum of n squared standard normal random variables and thus follows a $\chi^2(n)$-Distribution with n degrees of freedom and expected value n. The weighted sum has an unknown distribution (Castaño-Martínez and López-Blázquez, 2005), but we can still calculate its expected value as the sum of squared weights $\Sigma_{i=1}^{n} w_i^2$, which we use to scale the risk metric to an expected value of 1.
11. Which corresponds to a weighted version of the Euclidean Distance (De Maesschalck *et al.*, 2000).
12. The World Wealth Reports of Cap Gemini (1997-2015) cover investors from five different regions: Asia-Pacific, North America, Europe, Latin America and Africa/Middle East. All of them are subject to home bias to some degree, as their investments into assets from their home region range between 47 and 76 per cent of total investments.
13. Hoesli and Lizieri (2007) who report a real estate market size of US$8tn and a relative size of 20 per cent in relation to global equity market capitalization.
14. This feature goes hand in hand with heteroskedasticity or volatility clustering and relates to the fact that high volatility tends to be followed by high volatility. It is a well-documented feature of financial market returns (Bollerslev *et al.*, 1992).
15. Similar to the calculations used as a basis for Table VI, we take an ex ante-approach, i.e. the PRIX on day t is calculated using historical $\mu_{\mathrm{PRIP},t}$ and $\Sigma_{\mathrm{PRIP},t}$ up to day $t-1$. We have conducted a large variety of robustness checks for time windows of different lengths and threshold-quantiles, arriving at qualitatively similar results.

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